(2)

Paper I

Mechanics Oscillations and Properties of Matter

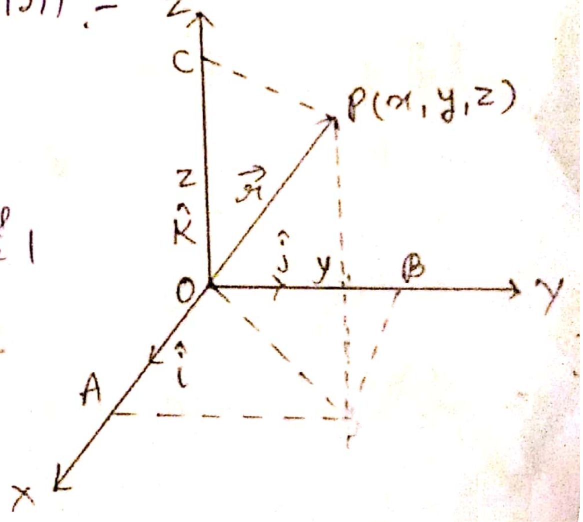
Unit-I

Topics:-

→ Cartesion.
→ Cylindrical and Spherical coordinate System.
→ Inertial and Non inertial frames of reference.
→ Uniformly rotating frame.
→ coriolis force and its applications.
→ motion under a central force.
→ Kepler's Law.
→ Effect of centrifugal and Coriolis forces due to earth's rotation
→ Centre of Mass (C.M.).
→ Lab and C.M. frame of reference
→ Motion of C.M. of system of particles subject to external forces.
→ Elastic and Inelastic collisions in one and two dimensions
→ Scattering angle in the laboratory frame of reference.
→ Conservation of linear and angular momentum.
→ Conservation of energy.

Cartesion

कार्तीय निर्देशांक पद्धति में किसी बिन्दु P की स्थिति समकोणिक X, Y, Z अक्षों के निर्देशांक (x, y, z) द्वारा व्यक्त की जाती है। यदि इन अक्षों के अनुदिश एकांक वेक्टर क्रमश: $\hat{i}$, $\hat{j}$ तथा $\hat{k}$ हैं तो बिन्दु P का स्थिति सदिश निम्नलिखित होगा :-



$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

यहाँ $OA = x$, $OB = y$ तथा $OC = z$ है।

$$v = \frac{d\vec{r}}{dt} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} + \hat{k}\frac{dz}{dt}$$

$$a = \frac{d\vec{V}}{dt} = \frac{d^2r}{dt^2} = \hat{i}\frac{d^2x}{dt^2} + \hat{j}\frac{d^2y}{dt^2} + \hat{k}\frac{d^2z}{dt^2}$$

→ Cylindrical Coordinate System

यहाँ P की स्थिति निर्देशांक (r, θ, z)

r= त्रिज्यीय स्थिति, θ कोणीय स्थिति, z= ऊर्ध्वाधर स्थिति

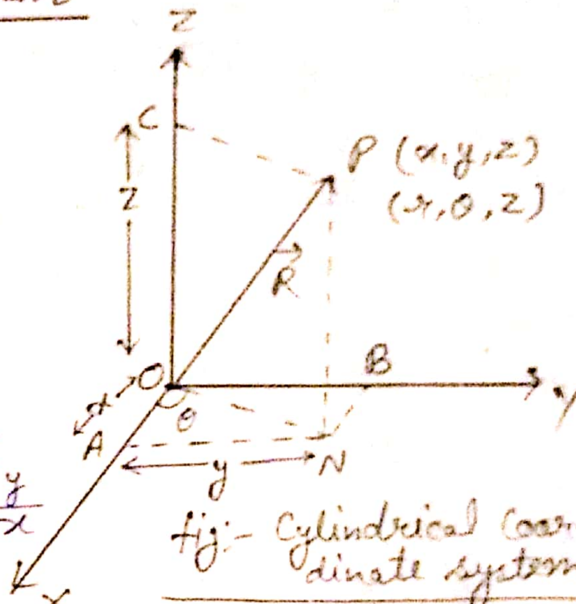


fig:- Cylindrical Coordinate system

$$x = OA = ON\cos\theta = r\cos\theta$$
$$y = OB = ON\sin\theta = r\sin\theta$$
$$z = OC = z$$

$$r = \sqrt{x^2+y^2}$$
$$\tan\theta = \frac{y}{x}$$
$$\theta = \tan^{-1}\frac{y}{x}$$

$$\vec{R} = \hat{i}(r\cos\theta) + \hat{j}(r\sin\theta) + \hat{k}z$$

$$\vec{V} = \frac{d\vec{R}}{dt} = \hat{i}\frac{d(r\cos\theta)}{dt} + \hat{j}\frac{d(r\sin\theta)}{dt} + \hat{k}\frac{dz}{dt}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{R}}{dt^2} = \frac{d}{dt}\left[\hat{i}\frac{d(r\cos\theta)}{dt} + \hat{j}\frac{d(r\sin\theta)}{dt} + \hat{k}\frac{dz}{dt}\right]$$

→ Spherical coordinate System

P की स्थिति निर्देशांक (r, θ, ϕ),

r= त्रिज्य स्थिति, θ = कोणीय स्थिति, φ= दिगंशी स्थिति

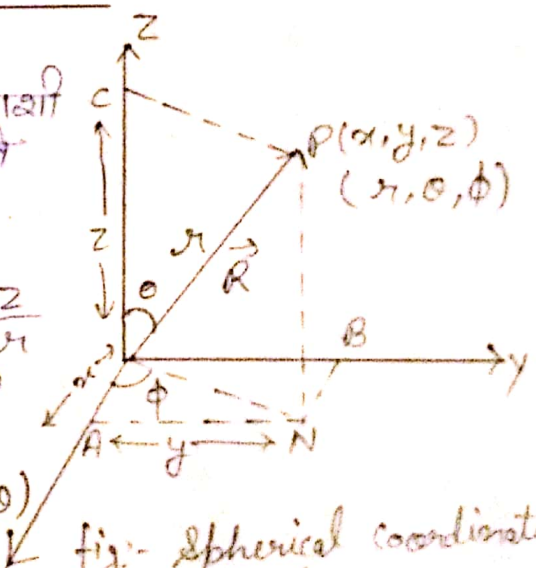


fig:- Spherical coordinate System

$$x = OA = ON\cos\phi = r\cos\phi \longrightarrow \cos\phi = \frac{x}{r}$$

$$\sin\phi = \frac{y}{r}$$

$$y = r\sin\phi$$

$$\sin\theta = \frac{z}{r}$$
$$z = r\sin\theta$$

$$\vec{R} = \hat{i}\,r\cos\phi + \hat{j}\,r\sin\phi + \hat{k}\,r\sin\theta$$

$$\vec{V} = \frac{d\vec{R}}{dt} = \hat{i}\frac{d(r\cos\phi)}{dt} + \hat{j}\frac{d(r\sin\phi)}{dt} + \hat{k}\frac{d(r\sin\theta)}{dt}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2R}{dt^2} = \frac{d}{dt}\left[\hat{i}\frac{d(r\cos\phi)}{dt} + \hat{j}\frac{d(r\sin\phi)}{dt} + \hat{k}\frac{d(r\sin\theta)}{dt}\right]$$

→ Inertial And Non Inertial frame of reference

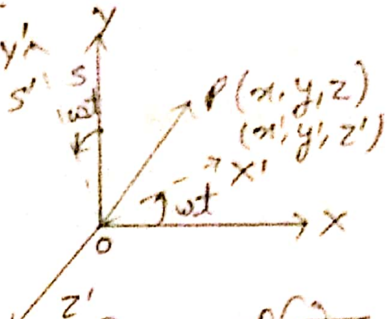
Inertial frame of reference:- An inertial frame of reference is a frame that is moving at a constant velocity.

Non Inertial frame of reference:- A non inertial frame of reference is just the opposite of Inertial frame of reference. It is a reference frame that is undergoing a nonzero acceleration.

→ Uniformly rotating frame

घूर्णन गति करता फ्रेम अजड़त्वीय फ्रेम होता है।

$$\left.\begin{aligned} x' &= x\cos\omega t + y\sin\omega t \\ y' &= -x\sin\omega t + y\cos\omega t \\ z' &= z \end{aligned}\right\} \text{—(1)}$$



चित्र:- घूर्णी फ्रेम

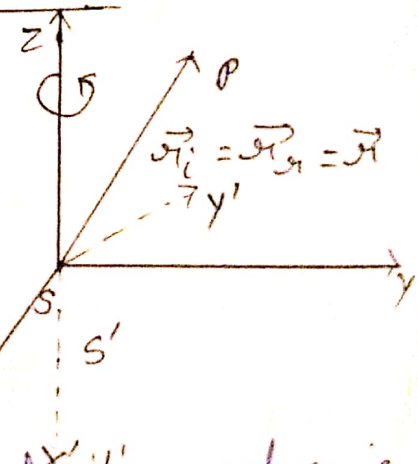
$$\left.\begin{aligned} x &= x'\cos\omega t - y'\sin\omega t \\ y &= x'\sin\omega t + y'\cos\omega t \\ z &= z' \end{aligned}\right\} \text{—(2)}$$

Ex:- माना एक बाल्टी में लगभग उसके आधे भाग तक पानी भरा है। बाल्टी के सापेक्ष स्थिर प्रेक्षक S को (या Inertial frame में) पानी का ~~बल~~ तल समतल दिखायी देता है। अब यदि पानी के अन्दर हाथ डालकर पानी को घुमाया जाये तो उसी प्रेक्षक S को पानी का तल परवलयाकार प्रतीत होगा, क्योंकि अब बाल्टी में भरा पानी, उस प्रेक्षक के सापेक्ष घूर्णी गति में है, अतः उस पर आभासी या छद्म बल लगता है।

→ Coriolis force and its applications

S and S' origin - O, Z axis concide (सम्पाती)
S' rotating frame around Z-axis at $t = 0$.
Both frames are concide.

S = Inertial frame.
S' = Non-Inertial frame.



Particle of mass at point P
observer from S and S' –

Let particle P is stationary w.r.t. S and position vector is $\vec{r}_i$ because origin is concided So, Position of P at S' is $\vec{r}_r$, where

$$\vec{r}_r = \vec{r}_i = \vec{r} \quad \text{—(1)}$$

frame S' is rotating with angular velocity w.r.t. S observer of S' observed particle moving in opposite dir^n with linear velocity $-\vec{\omega}\times\vec{r}$

S में प्रेक्षित कण का वेग $\qquad V_r = V_i - \vec{\omega} \times r$

$$\left(\frac{d\vec{r}}{dt}\right)_r = \left(\frac{d\vec{r}}{dt}\right)_i - \vec{\omega} \times r$$

{rotational frame में कण की velocity = Inertial में कण की velocity − प्रतीत velocity.

or $\left(\frac{dr}{dt}\right)_i = \left(\frac{dr}{dt}\right)_r + \vec{\omega} \times \vec{r}$ — ①

$$\vec{V_i} + \vec{\omega} + r = \vec{V_r} \text{ — ②}$$

$$\left(\frac{d\vec{V_i}}{dt}\right) = \left(\frac{d\vec{V_i}}{dt}\right)_r + \vec{\omega} \times \vec{V_i}$$

{eq. ① r के लिए इसी प्रकार V_i = velocity के Inertial frame के लिए।}

from eq ②:- $\left(\frac{d\vec{V_i}}{dt}\right)_i = \left[\frac{d}{dt}(\vec{V_r} + \underset{\mathrm{I}}{\vec{\omega}} \times \underset{\mathrm{II}}{r}) + \vec{\omega} \times (\vec{V_r} + \vec{\omega} \times \vec{r})\right]$

$$\Rightarrow \left(\frac{d\vec{V}}{dt}\right)_i = \left(\frac{d\vec{V_r}}{dt}\right)_r + \frac{d\vec{\omega}}{dt} \times r + \vec{\omega} \times \left(\frac{d\vec{r}}{dt}\right)_r + \vec{\omega} \times \vec{V_r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

or $\vec{a_i} = \vec{a_r} + \underbrace{\frac{d\vec{\omega}}{dt} \times r}_{0} + \underbrace{\vec{\omega} \times \vec{V_r}}_{\mathrm{I}} + \underbrace{\vec{\omega} \times \vec{V_r}}_{\mathrm{II}} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

because we Let that ω = constant So $\frac{d\vec{\omega}}{dt} = 0$

So $\vec{a_i} = \vec{a_r} + 2\vec{\omega} \times \vec{V_r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$ — ③

$$\Rightarrow \vec{a_r} = \vec{a_i} - 2\vec{\omega} \times \vec{V_r} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

multipling by m→

$$m\vec{a_r} = m\vec{a_i} - 2m\vec{\omega} \times \vec{V_r} - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\boxed{\vec{F_r} = \vec{F_i} + F_0}$$

$F_0 := -2m\vec{\omega} \times \vec{V_r} \Rightarrow$ Coriolis force

$-m\vec{\omega} \times (\vec{\omega} \times \vec{r}) \Rightarrow$ Centrifugal force

Applications :- ① पृथ्वी के वायुमण्डल में पाये जाने वाले विभिन्न वायु चक्रों (wind cycles) की व्याख्या Coriolis force द्वारा की जा सकती है।

② पृथ्वी की घूर्णन गति के कारण पृथ्वी पर किसी कण की सरल रेखीय गति, वक्राकार प्रतीत होती है। इसका कारण यह है कि पृथ्वी के उत्तरी अर्द्धगोलार्द्ध में गति करते कण को coriolis force दक्षिणावर्त कर देता है।

(3)

→ Cylindrical Coordinate System

$$\vec{R} = \hat{i}(r\cos\theta) + \hat{j}(r\sin\theta) + \hat{k}z$$

$$\vec{v} = \frac{d\vec{R}}{dt} = \hat{i}\frac{d(r\cos\theta)}{dt} + \hat{j}\frac{d(r\sin\theta)}{dt} + \hat{k}\frac{dz}{dt}$$

$$\vec{v} = \hat{i}\left[r\frac{d\cos\theta}{d\theta}\times\frac{d\theta}{dt} + \cos\theta\frac{dr}{dt}\right] + \hat{j}\left[r\frac{d\sin\theta}{d\theta}\times\frac{d\theta}{dt} + \sin\theta\times\frac{dr}{dt}\right] + \hat{k}\frac{dz}{dt}$$

$$\vec{v} = \hat{i}\left[-r\sin\theta\times\dot{\theta} + \cos\theta\,\dot{r}\right] + \hat{j}\left[r\cos\theta\times\dot{\theta} + \sin\theta\times\dot{r}\right] + \hat{k}\dot{z}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{R}}{dt^2} = +\hat{i}\left[-\left(r\frac{d\sin\dot{\theta}\theta}{d\dot{\theta}}\times\frac{d\dot{\theta}}{d\theta}\times\frac{d\theta}{dt} + \sin\theta\cdot\dot{\theta}\times\frac{dr}{dt} + \right.\right.$$

$$\left.\cos\theta\cdot\frac{d\dot{r}}{dr}\times\frac{dr}{dt} + \dot{r}\frac{d\cos\theta}{d\theta}\times\frac{d\theta}{dt}\right] + \hat{j}\left[r\frac{d\cos\dot{\theta}\theta}{d\dot{\theta}}\times\frac{d\dot{\theta}}{d\theta}\times\frac{d\theta}{dt} + \right.$$

$$\left.\cos\dot{\theta}\theta\times\frac{dr}{dt} + \dot{r}\frac{d\sin\theta}{d\theta}\times\frac{d\theta}{dt} + \sin\theta\times\frac{d\dot{r}}{dr}\times\frac{dr}{dt}\right]$$

$$\vec{a} = +\hat{i}\left[-\left(r\sin\theta\times d\ddot{\theta}\times\dot{\theta} + \cancel{\cos\theta}\sin\theta\cdot\dot{\theta}\times\dot{r} + \cos\theta\cdot\right.\right.$$

$$\left.\ddot{r}\times\dot{r} + \dot{r}(-\sin\theta\times\dot{\theta}\right] + \hat{j}\left[-r\sin\dot{\theta}\times\ddot{\theta}\times\dot{\theta} + \right.$$

$$\cos\dot{\theta}\theta\cdot\dot{r} + \dot{r}\cos\theta\times\dot{\theta} + \sin\theta\cdot\ddot{r}\times\dot{r}$$

→ Spherical Coordinate System

$$\vec{R} = \hat{i}\, r\cos\phi + \hat{j}\, r\sin\phi + \hat{k}\, r\sin\theta$$

$$\vec{v} = \frac{d\vec{R}}{dt} = \hat{i}\left[r\frac{d\cos\phi}{d\phi}\times\frac{d\phi}{dt} + \cos\phi\times\frac{dr}{dt}\right] + \hat{j}\left[r\cdot\frac{d\sin\phi}{d\phi}\times\frac{d\phi}{dt} + \sin\phi\times\frac{dr}{dt}\right] + \hat{k}\left[r\frac{d\sin\theta}{d\theta}\times\frac{d\theta}{dt} + \sin\theta\times\frac{dr}{dt}\right]$$

$$\vec{v} = \hat{i}\left[-r\sin\phi\times\dot{\phi} + \cos\phi\cdot\dot{r}\right] + \hat{j}\left[r\cos\phi\times\dot{\phi} + \sin\phi\times\dot{r}\right] + \hat{k}\left[r\cos\theta\times\dot{\theta} + \sin\theta\times\dot{r}\right]$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{R}}{dt^2} = \hat{i}\left[-\left(r\frac{d\sin\dot{\phi}}{d\dot{\phi}}\times\frac{d\dot{\phi}}{d\phi}\times\frac{d\phi}{dt} + \sin\phi\,\dot{\phi}\times\frac{dr}{dt}\right) + \cos\phi\times\frac{d\dot{r}}{dr}\times\frac{dr}{dt} + \dot{r}\times\frac{d\cos\phi}{d\phi}\times\frac{d\phi}{dt}\right] + \hat{j}\left[r\times\frac{d\cos\dot{\phi}}{d\dot{\phi}}\times\frac{d\dot{\phi}}{d\phi}\times\frac{d\phi}{dt} + \cos\dot{\phi}\times\frac{dr}{dt} + \sin\phi\times\frac{d\dot{r}}{dr}\times\frac{dr}{dt} + \dot{r}\times\frac{d\sin\phi}{d\phi}\times\frac{d\phi}{dt}\right] + \hat{k}$$

$$\left[r\times\frac{d\cos\dot{\theta}}{d\dot{\theta}}\times\frac{d\dot{\theta}}{d\theta}\times\frac{d\theta}{dt} + \cos\dot{\theta}\times\frac{dr}{dt} + \sin\theta\times\frac{d\dot{r}}{dr}\times\frac{dr}{dt} + \dot{r}\times\frac{d\sin\theta}{d\theta}\times\frac{d\theta}{dt}\right]$$

$$\vec{a} = \hat{i}\left[-\left(r\cos\dot{\phi}\times\ddot{\phi}\times\dot{\phi} + \sin\dot{\phi}\times\dot{r}\right) + \cos\phi\times\ddot{r}\times\dot{r} + \dot{r}\times(-\sin\phi)\times\dot{\phi}\right] + \hat{j}\left[r\times(-\sin\dot{\phi})\times\ddot{\phi}\times\dot{\phi} + \cos\dot{\phi}\times\dot{r} + \sin\phi\times\ddot{r}\times\dot{r} + \dot{r}\cos\phi\times\dot{\phi}\right] + \hat{k}\left[r\times(-\sin\dot{\theta})\times\ddot{\theta}\times\dot{\theta} + \cos\dot{\theta}\times\dot{r} + \sin\theta\times\ddot{r}\times\dot{r} + \dot{r}\times\cos\theta\times\dot{\theta}\right]$$

③

→ Motion Under a Central force

किसी कण पर लगने वाला वह बल जिसकी क्रिया-रेखा (line of action) सदैव एक स्थिर बिन्दु से होकर गुजरती है तथा जिसका परिमाण केवल उस बिन्दु से कण की दूरी पर निर्भर करता है, केन्द्रिय बल (Central force) कहलाता है।

→ Kepler's Law

(i) **Kepler's first Law:- (Orbital Law):-** प्रत्येक ग्रह सूर्य के चारों ओर दीर्घवृत्त कक्ष में गति करता है। जहाँ सूर्य focus की तरह होता है।

Derivation:- माना किसी ग्रह का द्रव्यमान m है तथा सूर्य का द्रव्यमान M है। m द्रव्यमान का सूर्य (M) के गुरुत्वीय क्षेत्र (Gravitation field) में गति करता है तो m द्रव्यमान के ग्रह पर दो प्रकार का बल लगता है।

(i) कैन्द्रीय बल (ii) केन्द्रीय बल।

केन्द्रिय बल सूर्य तथा ग्रह के द्रव्यमान के कारण लगता है जिसका मान:-

$\vec{F} = -\frac{GMm}{r^2}\hat{r}$ (where r is distance)

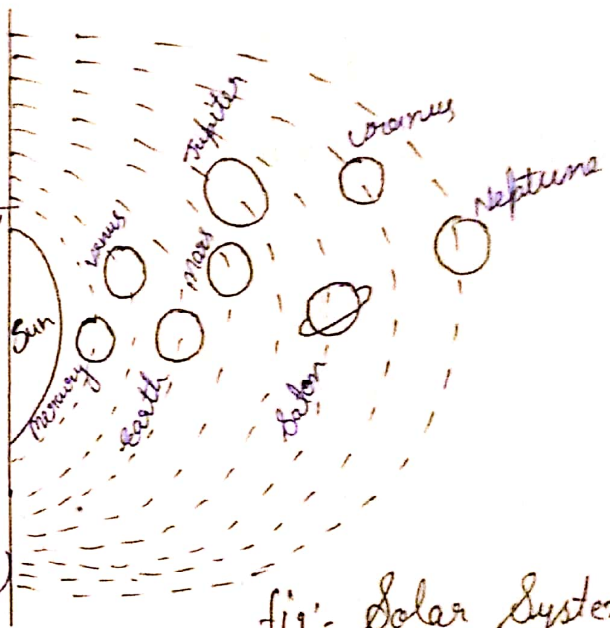
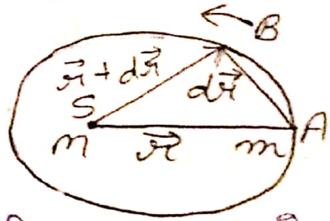


fig:- Solar System

तथा ग्रह पर लगने वाला कैन्द्रीय बल का मान:-

$\vec{F} = m \times$ radical acceleration



$F = m\left[a - r\omega^2\right]\hat{r}$

$F = m\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r}$

कैन्द्रीय बल तथा केन्द्रीय बल दोनों को बराबर होना चाहिए।

$\Rightarrow \frac{-GM\cancel{m}}{r^2}\hat{r} = \cancel{m}\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r}$

$\Rightarrow \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \frac{-GM}{r^2}$

$\Rightarrow r^3\frac{d^2r}{dt^2} - r^4\omega^2 = -GMr$ [r^3 का दोनों पक्षों में गुणा करने पर]:-

or $r^3\frac{d^2r}{dt^2} - h^2 = -GMr$ $[h = \omega r^2]$ — ③

Let $r = \frac{1}{u}$ $\therefore \omega = \frac{h}{r^2} = hu^2$ $\left[\because r = \frac{1}{u}\right]$

or $\frac{d\theta}{dt} = \frac{h}{r^2} = hu^2$

Now $\frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{u}\right) = \frac{-1}{u^2}\frac{du}{dt} = \frac{-1}{u^2}\frac{du}{d\theta}\cdot\frac{d\theta}{dt} = \frac{-1}{u^2}\frac{du}{d\theta}\cdot hu^2 = -h\frac{du}{d\theta}$

and $\frac{d^2r}{dt^2} = \frac{d}{dt^2}\left(-h\frac{du}{d\theta}\right)\left[\because \frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)\right] = -h\frac{d^2u}{d\theta^2}\cdot\frac{d\theta}{dt}$

$= -h\frac{d^2u}{d\theta^2}\cdot hu^2 = -h^2u^2\frac{d^2u}{d\theta^2}$

eq③ में $\frac{d^2r}{dt^2}$, r का मान रखने पर:-

$\frac{1}{u^3}\left(-h^2u^2\frac{d^2u}{d\theta^2}\right) - h^2 = -\frac{Gm}{u}$

$= -\frac{Gm}{u} = \frac{1}{u}\left(-h^2\frac{d^2u}{d\theta^2}\right) - h^2 = -\frac{Gm}{u}$

$\Rightarrow \frac{d^2u}{d\theta^2} + u = +\frac{Gm}{h^2}$ (h^2 से भाग कर पक्षान्तर करने पर फिर u से गुणा करने पर):-

$= \frac{d^2}{d\theta^2}\left(u - \frac{Gm}{h^2}\right) + \left(u - \frac{Gm}{h^2}\right) = 0$ — ④

$\Rightarrow \frac{Gm}{h^2}$ is constant So. $\frac{d^2}{d\theta^2}\left(u - \frac{Gm}{h^2}\right) = 0$

Let solution of eq④ is:- $\Rightarrow u - \frac{Gm}{h^2} = A\cos\theta$

$\Rightarrow u = \frac{Gm}{h^2} + A\cos\theta$ [$\frac{h^2}{Gm}$ से गुणा करने पर]:-

$\Rightarrow u\times\frac{h^2}{Gm} = 1 + \frac{h^2}{Gm}A\cos\theta \Rightarrow \frac{h^2/Gm}{r} = 1 + \frac{h^2}{Gm}A\cos\theta$ $\left[\because r = \frac{1}{u} \Rightarrow u = \frac{1}{r}\right]$

शंकु परिच्छेद $\frac{l}{r} = 1 + e\cos\theta$ से तुलना करने पर:-

Findings:- $e = \frac{h^2A}{Gm}$

$\Rightarrow l = h^2/Gm$

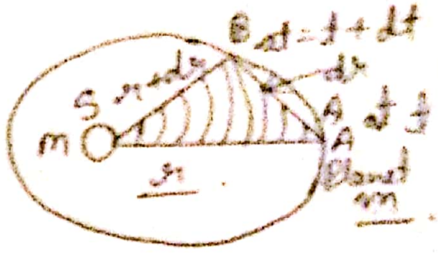
Conclusion:- $e<1$ होता है इसलिए ग्रह की गति दीर्घवृत्त होगी।

(If $e<1$ than it indicats that it is ellipse so motion of planet arround the sun is in elliptical path)

④

⇒ Kepler's Second Law (Area Law (क्षेत्रफल का नियम)):-

Derivation:- At A → t
after dt time at B → t + dt



Planet → Linear velocity = V
Angular velocity = ω $V = \omega r$ — ①

Angular momentum at point $L = mr \times V$
$= mr \times \omega r = mr^2\omega$ — ②
$= I\omega$

$V = \frac{L}{mr}$

$\triangle SAB$ Area $dA = \frac{1}{2} \times \vec{r} \times d\vec{r} \Rightarrow \frac{dA}{dt} = \frac{1}{2} \times \vec{r} \times \frac{d\vec{r}}{dt}$

$\frac{dA}{dt} = \frac{1}{2} \times r \times V$ — ③ $\Rightarrow \frac{dA}{dt} = \frac{1}{2} \times \cancel{r} \times \frac{L}{m\cancel{r}}$

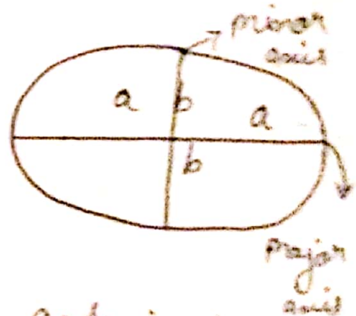
Eindinas:- $\frac{dA}{dt} = \frac{L}{2m}$ — ④ $\left[\frac{L}{2m} = \frac{\cancel{m}r^2\omega}{2\cancel{m}} = \frac{r^2\omega}{2} = \frac{h}{2}\right]$

Conclusion:- L व m नियत है अर्थात $\frac{dA}{dt}$ भी नियत है, यही केपलर का द्वितीय नियम है।

⇒ Kepler's Third Law:-

किसी भी ग्रह के आवर्तकाल का वर्ग उस ग्रह के दीर्घवृत्त के अर्द्ध दीर्घ अक्ष के त्रिघात के अनुक्रमानुपाती होता है। $\boxed{T^2 \propto a^3}$

Derivation:- If Semimajor axis is a and Semiminor axis is b then:-



a: Semimajor axis
b = Semiminor axis

$a = \frac{l}{1-e^2}$ and $b = a = \frac{l}{1-e^2}$

दोनों पक्षों में $\sqrt{1-e^2}$ से गुणा करने पर:-

$b = a\sqrt{1-e^2} = \frac{l}{\sqrt{1-e^2}}$

Squaring both side:- $b^2 = \frac{l^2}{(1-e^2)} \Rightarrow b^2 = l \times \frac{l}{1-e^2}$

$\Rightarrow b^2 = la \Rightarrow l = \frac{b^2}{a} = \frac{h^2}{GM}$ (According to first Law):-

$\Rightarrow b^2 = \frac{h^2 a}{GM}$ — ①   सूर्य के परित: ग्रह का आवर्तकाल

(Timeperiod the planet around the Sun):-

$T = \frac{\text{दीर्घवृत्त का क्षेत्रफल}}{\text{क्षेत्रीय वेग}}$

क्षेत्रीय वेग $= \frac{L}{2m} = \frac{h}{2} \left[\frac{L}{2m} = \frac{\cancel{m}r^2\omega}{2\cancel{m}} = \frac{h}{2}\right]$

$$T = \frac{\pi ab}{\frac{h}{2}} = \frac{2\pi ab}{h}$$ Squaring both side:-

$$T^2 = \frac{4\pi^2 a^2 b}{h^2} = \frac{4\pi^2 a^2}{\cancel{h^2}} \times \frac{\cancel{h^2} \times a}{GM} \left[\text{Eq ① से } b^2 = \frac{h^2 \times a}{GM} \right]$$

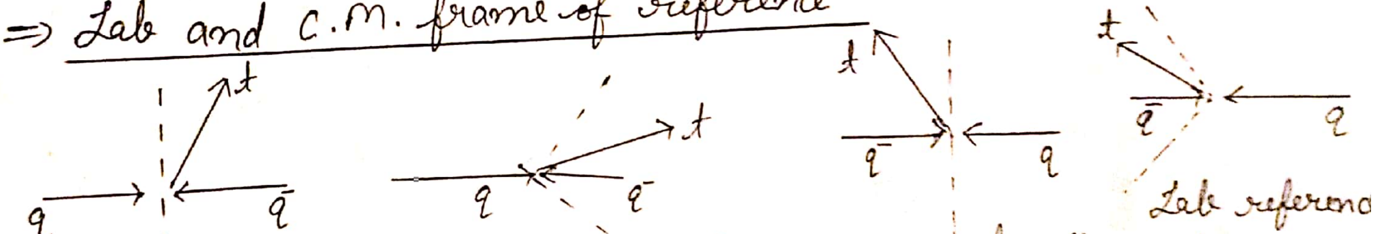
$$T^2 = \frac{4\pi^2 a^3}{GM} \left[\frac{4\pi^2 a^3}{GM} = \text{constant} \right]$$ Conclusion:- $\boxed{T^2 \propto a^3}$

⇒ Effect of Centrifugal and Coriolis force due to earth's rotation:- Deflection of an object due to the Coriolis force is called the Coriolis effect. The centrifugal force acts outwards in the radial direction and its is proportional to the distance of the body from the axis of the rotating frame. These additional forces are termed inertial forces, fictitious forces or pseudo forces.

⇒ Centre of Mass (C.M.) ⇒ किसी पिण्ड अथवा कणों के निकाय का द्रव्यमान केंद्र वह बिंदु होता है, जिस पर पिण्ड अथवा निकाय का सम्पूर्ण द्रव्यमान केन्द्रित माना जा सकता है।

$$\boxed{\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}}$$

⇒ Lab and C.M. frame of reference



C.M. reference frame — Lab reference frame — CM reference frame — Lab reference frame

Relationship between center of mass and laboratory frames of reference:-

Definition of center of mass R_{cm}

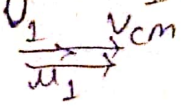
$$m_1 r_1 + m_2 r_2 = (m_1 + m_2) R_{cm}$$

$$m_1 \vec{r}_1 + m_2 \vec{r}_2 = (m_1 + m_2) \vec{R}_{cm}$$

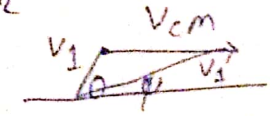
$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) V_{cm} = m_1 V_1 + m_2 V_2$$

In our case:- $V_{cm} = \frac{m_1}{m_1 + m_2} u_1 = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$

$u_1 = U_1 + V_{cm}$



$v_1 = V_1 + V_{cm}$



⇒ Motion of C.M. of System of particles subject to external forces.

n particles $\xrightarrow{}$ m_1

F_{ext} $\longrightarrow m_2$

$\longrightarrow m_3$

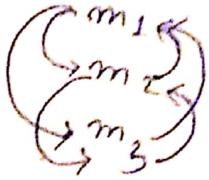
⋮

$\longrightarrow m_n$

$m_1, m_2, m_3 \ldots m_n$ में F_{ext} लगाने के बाद motion में आ जाता है।

$m_1, m_2 \ldots m_3$ के कारण जो force m_1 पर लग रहा है वही force F_{int} है।

ith particle :- $\boxed{F_i = F_{ext} + F_{int}}$ — (1)



$F_{int} = F_{i_1} + F_{i_2} + F_{i_3} + F_{i_4} + F_{i_5} + \cdots F_{in}$

$$\boxed{F_{int} = \sum_{j=1}^{n} F_{ij}} \quad -(2) \qquad \boxed{F_i = F_{ext} + \sum_{j=1}^{n} F_{ij}} \quad -(3)$$

n-कण :- $F = F_1 + F_2 + F_3 + \ldots F_n$

$$F = \left(F_1 ext + \sum_{\substack{j=1 \\ i \neq j}}^{n} F_{ij}\right) + F_2 + \sum_{\substack{j=1 \\ i \neq j}}^{n} F_2$$

$$\boxed{F = \sum_{i=1}^{n} F_{i\,ext} + \sum_{\substack{j=1 \\ i \neq j}}^{n} F_{ij(int)}}$$ final eqn.

$F = ma$

$$m\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i$$

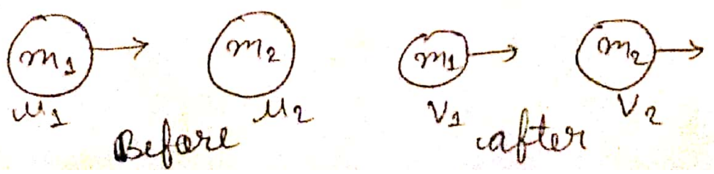
$$\boxed{a_{cm} = \frac{\sum_{i=1}^{n} F_i}{m}}$$

⇒ Elastic and Inelastic collisions in one and two dimensions:-

when two particles come close to each other due to Interaction b/w them, there motion changes, this phenomena is called collision.

In absence of Internal force, Linear momentum remains conservation.

(1) Elastic Collision (2) Inelastic Collision.



As per conservation of linear momentum:-

$$\left.\begin{aligned} &\Rightarrow m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2 \\ &\Rightarrow m_1 u_1 - m_1 V_1 = m_2 V_2 - m_2 u_2 \\ &\Rightarrow m_1 (u_1 - V_1) = m_2 (V_2 - u_2) \end{aligned}\right\} \text{ — } 1$$

As per conservation of Energy:-

$$\Rightarrow \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

$$\Rightarrow \cancel{\frac{1}{2}} (m_1 u_1^2 + m_2 u_2^2) = \cancel{\frac{1}{2}} (m_1 V_1^2 + m_2 V_2^2)$$

$$\Rightarrow m_1 u_1^2 - m_1 V_1^2 = m_2 V_2^2 - m_2 u_2^2$$

$$\Rightarrow m_1 (u_1^2 - V_1^2) = m_2 (V_2^2 - u_2^2) \text{ — } 2$$

$$\Rightarrow m_1 (u_1 - V_1)(u_1 + V_1) = m_2 (V_2 - u_2)(V_2 + u_2)$$

from eq. 1 $m_1 (u_1 - V_1) = m_2 (V_2 - u_2)$:-

$$\Rightarrow m_2 \cancel{(V_2 - u_2)} (u_1 + V_1) = m_2 \cancel{(V_2 - u_2)} (V_2 + u_2)$$

$$(u_1 - u_2) = (V_2 - V_1) \text{ — } 3$$

Relative velocity before collision = R.V. after collision.

from eq 3 :-

$$V_2 = u_1 - u_2 + V_1$$

Putting the value of V_2 in eq 1

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 (u_1 - u_2 + V_1)$$

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 u_1 - m_2 u_2 + m_2 V_1$$

$$\Rightarrow (m_1 - m_2) u_1 + 2 m_2 u_2 = m_1 V_1 + m_2 V_1$$

$$\Rightarrow (m_1 - m_2) u_1 + 2 m_2 u_2 = (m_1 + m_2) V_1 \text{ — } 4$$

Value of V_1 in eq 3 :- $V_1 = V_2 - u_1 + u_2$

Putting the value of V_1 in eq 1 :-

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 (V_2 - u_1 + u_2) + m_2 V_2$$

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 V_2 - m_1 u_1 + m_1 u_2 + m_2 V_2$$

$$\Rightarrow m_1 u_1 + m_2 u_2 + m_1 u_1 - m_1 u_2 = (m_1 + m_2) V_2$$

$$\Rightarrow 2 m_1 u_1 + (m_2 - m_1) u_2 = (m_1 + m_2) V_2$$

$$V_2 = \frac{2 m_1 u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2} u_2 \text{ — } 5$$

Case (1):- If $m_1 = m_2$:-

Than from eq (4) $V_1 = 0 + u_2 \Rightarrow V_1 = u_1$

from eq (5) $V_2 = u_2$

Means of mass of particles are equal than velocity of Interchange after collision.

Case (2):- If $u_2 = 0$ and $m_1 = m_2$ than $V_1 = 0$

$V_2 = u_1$

⇒ **Scattering angle in the laboratory frame of reference**

Laboratory frame:- (प्रयोगशाला निर्देश तंत्र) केन्द्रीय बलों के अन्तर्गत दो कणों के निकाय की गति का अध्ययन इनमें से भारी कण को स्थिर मूलबिन्दु मानकर दूसरे कण के स्थिति सदिश पर निकाय के समानीत द्रव्यमान μ के कण की गति द्वारा कर सकते हैं। इस निर्देशांक निकाय को प्रयोगशाला निर्देशांक निकाय (Laboratory frame) कहते हैं।

चित्र के अनुसार माना कि m_1 व m_2 द्रव्यमान कणों का किसी स्वेच्छिक मूलबिन्दु O के सापेक्ष स्थिति सदिश क्रमशः $\vec{r}_1$ व $\vec{r}_2$ हैं तथा m_1 द्रव्यमान कण का m_2 के सापेक्ष स्थिति सदिश $\vec{r}$ हो, तो प्रयोगशाला निर्देशांक निकाय में इस निकाय की गति को $\vec{r} = \vec{r}_1 - \vec{r}_2$ पर एक काल्पित द्रव्यमान μ की गति द्वारा प्रदर्शित किया जाता है, जहाँ निकाय का समानीत द्रव्यमान

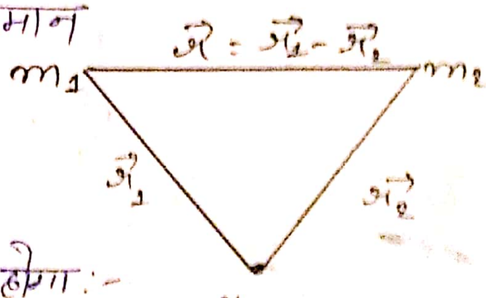


fig:-

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad -(1)$$

होगा तथा निकाय के गति का समीकरण निम्न होगा:-

$$\boxed{\mu \frac{d^2\vec{r}}{dt^2} = \vec{F}} \quad -(2)$$

जहाँ $\vec{F}$ द्रव्यमान कण m_1 पर लगने वाला केन्द्रीय बल है। समीकरण (2) को सदिश $\vec{r}$ के लिए हल करके $\vec{r}_1$ व $\vec{r}_2$ के मान ज्ञात कर सकते हैं।

⇒ Conservation of Linear momentum:-

$\vec{P} = m\vec{v}$ — (1) Second law of Newton.

$\vec{F} = \frac{d\vec{P}}{dt}$

$F = \frac{d}{dt} m\vec{v} = m\vec{a}$ — (2)

If $\vec{F} = 0$ $\quad \frac{d\vec{P}}{dt} = 0 \quad \boxed{\vec{P} = \text{constant}}$

for N particle System:- $\vec{P_1} + \vec{P_2} + \ldots \vec{P_n} = \text{constant}$

$$\boxed{\sum_{i=1}^{i=n} P_i = \text{constant}}$$

⇒ Application of Conservation of Linear momentum:-

1. Firing of bullet from a gun.
2. Fission of nucleus.
3. Propulsion of Boat and aeroplane.
4. Rocket.

⇒ Conservation of Angular momentum:-

Angular momentum $\vec{L} = \vec{r} \times \vec{P}$

change in $\vec{L}$ w.r. to time.

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{P})$$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{P} + \vec{r} \times \frac{d\vec{P}}{dt}$$

$$\frac{d\vec{L}}{dt} = \cancel{\vec{v} \times m\vec{v}} + \vec{r} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = 0 + \vec{r} \times \vec{F}$$

$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$ [F → around radius is τ = Torque,

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

If $\vec{\tau} = 0$ then $\frac{d\vec{L}}{dt} = 0$

$$\boxed{\frac{d\vec{L}}{dt} = \text{constant.}}$$

(7) Conservation of Energy:-

$$W_{12} = \sum_{i=1}^{n} \int_1^2 \vec{F_i} \cdot \vec{dr_i} = \sum_{i=1}^{n} \int_1^2 \vec{F}_{i\,ext} \cdot \vec{dr_i} + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \int_1^2 \vec{F_{ij}} \cdot \vec{dr_i}$$

ith particle

where $F_i = m_i \frac{d\vec{V_i}}{dt}$ and $\vec{dr_i} = \vec{V_i}\,dt$.

$$\therefore W_{12} = \sum_{i=1}^{n} \int_1^2 m_i \frac{d\vec{V_i}}{dt} \cdot \vec{V_i}\,dt = \sum_{i=1}^{n} \int_1^2 m_i \vec{V_i} \cdot d\vec{V_i} \quad -①$$

$$= \left[\sum_{i=1}^{n} \frac{1}{2} m_i v_i^2 \right]_1^2 = K_2 - K_1$$

$$\vec{F}_{i\,ext} = -\left[\hat{i}\frac{\partial U_i}{\partial x_i} + \hat{j}\frac{\partial U_i}{\partial y_i} + \hat{k}\frac{\partial U_i}{\partial z_i} \right] \Rightarrow \vec{F_{ij}} = -\left[\hat{i}\frac{\partial U_{ij}}{\partial x_i} + \hat{j}\frac{\partial U_{ij}}{\partial y_i} + \hat{k}\frac{\partial U_{ij}}{\partial z_i} \right]$$

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq 1}}^{n} \int_1^2 \vec{F_{ij}} \cdot \vec{dr_i} = \frac{1}{2} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq 1}}^{n} \int_1^2 \left(\vec{F_{ij}} \cdot \vec{dr_i} + \vec{F_{ji}} \cdot \vec{dr_j} \right)$$

$$= -\frac{1}{2} \sum_{i=1}^{n} \sum_{\substack{i=1 \\ i\neq j}}^{n} \int_1^2 dU_{ij}$$

eq ① से:-

$$W_{12} = \sum_{i=1}^{n} \int_1^2 -\frac{\partial U_i}{\partial r_i} \hat{r}_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{\substack{i=1 \\ j\neq 1}}^{n} \int_1^2 dU_{ij}$$

$$W_{12} = -\left[U\right]_1^2 = U_1 - U_2 \quad -②$$

~~from~~ eq. ① and ②:-

$$K_2 - K_1 = U_1 - U_2$$

$$\boxed{K_1 + U_1 = K_2 + U_2 = E \text{ (Constant)}}$$

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